Public Investment in a Production Network: Aggregate and Sectoral Implications*

Alessandro Peri — University of Colorado, Boulder

Omar Rachedi — Esade Business School, Universitat Ramon Llull

Iacopo Varotto — Banco de España

Abstract

Aggregate and sectoral effects of public investment crucially depend on the interaction between the output elasticity to public capital and intermediate inputs. We uncover this fact through the lens of a New Keynesian production network. This setting doubles the socially optimal amount of public capital relative to the one-sector model without intermediate inputs, leading to a substantial amplification of the public-investment multiplier. We also document novel sectoral implications of public investment. Although public investment is concentrated in far fewer sectors than public consumption, its effects are relatively more evenly distributed across industries. We validate this model implication in the data.

Key Words: Sectoral Heterogeneity, Input-Output Matrix, Public Capital.

JEL Classification Codes: E31, E32, E52.

---

*Email: alessandro.peri@colorado.edu, omar.rachedi@esade.edu, iacopo.varotto@bde.es.

We thank the Editor (Olivier Coibion), three anonymous referees, Henrique S. Basso, Hafedh Bouakez, Christian Höynck, Emiliano Santoro, and presentation participants at ESCP Business School, the ZEW Public Finance Conference in Mannheim, the Workshop in Empirical and Theoretical Macroeconomics in London, the Catalan Economic Society Congress in Barcelona, and the Annual CEPR MEF Symposium on Rethinking Macroeconomic Policy in Times of Turmoil in Aegina for useful comments and suggestions. Omar Rachedi acknowledges the support received from the Spanish Ministry of Science, Innovation and Universities through the project PID2020-120013GB-I00. This paper was previously circulated under the title "The Public Investment Multiplier in a Production Network". The opinions in this paper are those of the authors and do not necessarily reflect the views of the Banco de España or the Eurosystem.

1 Introduction

In the aftermath of the Covid pandemics, governments turned to massive public-investment projects, such as the $1.2 trillion Infrastructure Investment and Jobs Act in the U.S. and the €800 billion Next Generation EU in Europe. Policy-makers motivate these packages with the need to strengthen supply chains and foster the development of specific industries.[1] However, these mechanisms are missing in the workhorse analysis of public investment, which hinges on one-sector models. To fill this gap, this paper studies the implications of public investment through a New Keynesian production network. We find that the aggregate and sectoral effects of public investment crucially depend on the interaction between the output elasticity to public capital and the presence of intermediate inputs.

To study the production-network propagation of public-investment shocks, we build a sticky-price model with heterogeneous sectors that are connected by an Input-Output matrix. The government finances an exogenous stream of public spending on sectoral goods with lump-sum taxes. Public investment accumulates to the stock of public capital subject to time-to-build and time-to-spend delays, as in Leeper et al. (2010b) and Ramey (2021). Public capital enhances the productivity of final-good technologies, to an extent which varies across industries.

In the quantitative analysis, we consider an economy with 55 sectors and calibrate it with information from the U.S. Input-Output Tables. To discipline the heterogeneous effect of public capital across industries, we provide novel estimates of the elasticity of gross output to public capital at the sectoral level. We extend Bouakez et al. (2017)'s estimation to a heterogeneous cointegrated panel setting, and recover the sector-specific elasticities by regressing the logarithm of

[1] See https://www.whitehouse.gov/bipartisan-infrastructure-law/ and https://ec.europa.eu/info/strategy/recovery-plan-europe_en.

utilization-adjusted sectoral TFP on the logarithm of public capital. We find an average elasticity of 0.0575, at the low end of the estimates derived in the literature. This figure conceals substantial heterogeneity: the elasticity varies from 0 for the pipeline transportation industry up to 0.1363 for computer manufacturing.

Our first contribution is to measure to what extent sectoral heterogeneity and inter-linkages alter the aggregate effects of public investment.[2] The production-network economy implies a long-run present-value public-investment multiplier of 2.12, which is 68% larger than in the one-sector model without intermediate inputs.[3] Crucially, the amplification doubles that of the public-consumption multiplier (Bouakez et al., 2023),[4] and is also substantial at short horizons. Although public consumption spurs relatively more GDP on impact (Boehm, 2020; Ramey, 2021), the production network closes the gap between the public-investment and public-consumption multipliers after 6 quarters, significantly faster than the 7 years required by the one-sector model. Thus, our model can reconcile the facts

[2]We consider a shock to aggregate government spending that raises government purchases across industries in a way that preserves the sectoral composition of public investment observed in the data.

[3]Throughout the paper, we use interchangeably the terms "one-sector economy without intermediate inputs" and "one-sector economy". However, our results emphasize that the amplification result hinges on comparing our multi-sector economy with one without intermediate inputs, whose absence matters much more than the precise number of sectors.

[4]This finding holds in an comprehensive battery of robustness checks that extend the baseline economy to include features such as distortionary taxes (Leeper et al., 2010a), sticky wages (Erceg et al., 2000), an investment network (Vom Lehn and Winberry, 2022), and durable consumption (Boehm, 2020).

that the stimulus effects of public investment are limited in the short run (Boehm, 2020) but become significant after two years (Ilzetzki et al., 2013).

The amplification of the public-investment multiplier fully stems from the interaction between the output elasticity to public capital and intermediate inputs. When public capital is unproductive, public investment barely alters output. When it is productive but absent intermediate inputs, the multipliers in the multi-sector and one-sector economies coincide. We characterize analytically the mechanism boosting the response of aggregate output to public investment. In presence of a production network, public investment benefits firms not only directly, but also indirectly: by enhancing the efficiency in the provision of intermediate inputs, public investment curtails firms' costs, boosting production. This mechanism yields larger public-investment multipliers if upstream sectors feature high public-capital elasticities. However, sectors' positions in the network barely correlate with their public-capital elasticities, which implies that heterogeneity in this dimension does not play a sizable role in the propagation of public investment.

Our amplification result relates to the way in which intermediate inputs alter the optimal level of public capital, as output reacts relatively more to public-investment shocks when public capital is inefficiently low (Ramey, 2021). Consistently with this logic, we find that the production network doubles the optimal stock of public capital relative to the one-sector economy, and shifts the welfare costs of inefficient levels: while welfare losses in the one-sector economy mainly come when public capital is inefficiently high, the opposite applies in our model. Thus, inter-sectoral linkages exacerbate the costs of low levels of public capital.

Our second contribution is to uncover novel sectoral implications of government spending. The interaction between intermediate inputs and the output elasticity to public capital plays a key role in distributing the output gains of public investment more evenly across sectors compared to those of public consumption, despite the stark concentration of public investment in just few sectors.[5] This is because the Input-Output matrix magnifies the positive effects of public capital across the production network, as sectors may *indirectly* benefit from the higher efficiency in the provision of intermediate inputs, even when they do not *directly* contribute to the production of public-investment goods. As a result, the output gains propagate to a wider pool of industries. Conversely, since public consumption is not productive, its benefits mainly accrue to those sectors that are *direct* recipients of this type of spending.

Finally, we empirically validate the sectoral implications of the model by testing whether sectors' direct contributions to government spending matter less for the sectoral responses to public-investment shocks than for public-consumption shocks. We adapt the estimation strategy of Ramey and Zubairy (2018) by extending the linear projection method of Jordà (2005) to a panel setting. We regress sectoral value added on the interaction between aggregate defense investment expenditures and the associated sectoral contributions. Aggregate public spending is instrumented with both the military-spending news variable of Ramey (2011) and the timing restriction of Blanchard and Perotti (2002). We also consider an analogous regression for public consumption. We find that the coefficients on the interaction terms are positive and highly statistically significant, with the estimate of the public-consumption interaction doubling that of

[5] Just three industries account for 78% of public investment. For public consumption, to derive a total joint share of 78% requires summing over the largest 15 recipient industries. However, the ratio between the standard deviation of the sectoral distribution of the aggregate multiplier—which informs on how one additional dollar of the aggregate multiplier is distributed across sectors—and sectors' contributions to government spending equals 0.4 for public investment, and it exceeds unity for public consumption.

public investment. Notably, the empirical estimates on how the sectoral distribution of the aggregate multipliers vary with sectors' contributions to public spending are remarkably in line with the quantitative predictions of the model.

We add to the literature on the aggregate effects of public investment (Baxter and King, 1993; Leeper et al., 2010b; Leduc and Wilson, 2013; Bouakez et al., 2017, 2020; Boehm, 2020; Ramey, 2021; Malley and Philippopoulos, 2023), by showing that the public-investment multiplier is substantially amplified in a production network.[6] This result sheds novel insights on the values of the output elasticity to public capital that should be used in one-sector models. We emphasize the difference between gross-output-based and value-added-based elasticities, and argue that estimates that recover the elasticity from a technology in gross output terms should be mapped into one-sector economies by adjusting for the share of intermediate inputs. In other words, an average elasticity of 0.0575 in our production-network yields a multiplier that can be reproduced by the one-sector model with an elasticity of $0.1105 = 0.0575/(1 - 0.4779)$, where 0.4779 is the share of intermediate inputs in aggregate gross output. Finally, we provide novel predictions on the sectoral implications of public investment. This dimension has been neglected until now, given the prominent use of one-sector models to study public spending.

Our work builds on the literature that studies how business cycle fluctuations are shaped by sectoral heterogeneity and linkages (Acemoglu et al., 2012; Baqaee and Farhi, 2019; Pasten et al., 2020). In this context, we complement the body of work that looks at the propagation of fiscal shocks across heterogeneous sectors (Acemoglu et al., 2016; Baqaee and Farhi, 2018; Bouakez et al., 2022, 2023;

[6] Our emphasis on the relevance of sectoral heterogeneity complements Cai and Roulleau-Pasdeloup (2023), where household heterogeneity amplifies the public-investment multiplier.

Cox et al., 2022; Proebsting, 2022). While these papers focus solely on the effects of *public-consumption* shocks, we show to what extent—and through which channels—a production network alters the effects of *public-investment* shocks.

2 Model

The economy consists of a unit mass of identical infinitely-lived households and a finite number of heterogeneous sectors, indexed by $s \in \{1, \ldots, S\}$. The government consists of a monetary authority, which sets the nominal interest rate with a Taylor rule, and a fiscal authority, which sets a lump-sum tax on the households to finance exogenous streams of public consumption and public investment. The model features ingredients that are key to generate a realistic short-run transmission of government spending, such as the presence—and heterogeneity—of price rigidities (Hall, 2009; Bouakez et al., 2023) and limited sectoral reallocation of labor and capital (Bouakez et al., 2022; Proebsting, 2022).

2.1 Household

The representative household has preferences over streams of private consumption, C_t, and labor, N_t, such that the present value of its life-time utility equals

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \theta \frac{N_t^{1+\eta}}{1+\eta} \right], \tag{1}$$

where β is the time discount factor, σ captures the risk aversion, η is the inverse of the Frisch elasticity, and θ is a labor disutility shifter. The budget constraint

$$P_{C,t}C_t + P_{I,t}I_t + T_t + B_t = W_t N_t + R_{K,t}K_t + R_{t-1}B_{t-1} + F_t \tag{2}$$

posits that every period the household purchases the private-consumption good at price $P_{C,t}$, the private-investment good I_t at price $P_{I,t}$, and incurs in a nominal lump-sum tax, T_t. The household earns labor income, $W_t N_t$, and capital income, $R_{K,t}K_t$, where W_t is the aggregate nominal wage, K_t denotes the stock of private capital, and $R_{K,t}$ is its nominal return rate. The household also invests in one-period bonds, B_t, that yield the nominal rate R_t, and receives firms' profits, F_t.

Private capital depreciates at the rate δ_K and its law of motion is subject to investment adjustment costs, captured by the parameter Ω, such that

$$K_{t+1} = (1 - \delta_K) K_t + I_t \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right]. \tag{3}$$

To capture the limited reallocation of labor and capital across sectors at business cycle frequencies (Lee and Wolpin, 2006; Lanteri, 2018), we assume that aggregate labor and the aggregate stock of private capital are CES aggregators of sectoral labor flows $N_{s,t}$ and private capital $K_{s,t}$ supplied to sector s

$$N_t = \left[\sum_{s=1}^{S} \omega_{N,s}^{-\frac{1}{\nu_N}} N_{s,t}^{\frac{1+\nu_N}{\nu_N}} \right]^{\frac{\nu_N}{1+\nu_N}} \quad \text{(4a)} \qquad K_t = \left[\sum_{s=1}^{S} \omega_{K,s}^{-\frac{1}{\nu_K}} K_{s,t}^{\frac{1+\nu_K}{\nu_K}} \right]^{\frac{\nu_K}{1+\nu_K}}. \quad \text{(4b)}$$

Here, $\omega_{N,s}$ and $\omega_{K,s}$ represent the sectoral weights, and ν_N and ν_K capture the elasticity of substitutions of labor and capital across sectors, respectively. Reallocation frictions (which are active insofar $\nu_N, \nu_K < \infty$) generate heterogeneous sectoral nominal wages $W_{s,t}$ and sectoral nominal returns of private capital $R_{K,s,t}$, that yield the following expressions for the aggregate nominal wage and the aggregate nominal return of private capital:

$$W_t = \left[\sum_{s=1}^{S} \omega_{N,s} W_{s,t}^{1+\nu_N} \right]^{\frac{1}{1+\nu_N}} \quad \text{(5a)} \qquad R_{K,t} = \left[\sum_{s=1}^{S} \omega_{K,s} R_{K,s,t}^{1+\nu_K} \right]^{\frac{1}{1+\nu_K}}. \quad \text{(5b)}$$

2.2 Firms

Each sector is operated by two layers of firms: a continuum of monopolistic producers that assemble different varieties of the sectoral good, and competitive wholesalers that bundle the varieties into the final sectoral good.

Sectoral goods are sold to competitive private- and public-consumption retailers, private- and public-investment retailers, and intermediate-input retailers which produce the final private and public consumption, the final private and public investment, and the intermediate inputs used by all sectors, respectively.

The output of private-consumption and investment retailers is sold to households, the output of intermediate-input retailers is sold to producers, and the output of public-consumption and investment retailers is sold to the government.

2.2.1 Producers

Each sector s is populated by a unit mass of homogeneous monopolistic producers, indexed by $i \in [0,1]$, which assemble different varieties of the sectoral good, $Z^i_{s,t}$, according to the Cobb-Douglas technology

$$Z^i_{s,t} = \left({N^i_{s,t}}^{\alpha_{N,s}} {K^i_{s,t}}^{1-\alpha_{N,s}} \right)^{1-\alpha_{H,s}} {H^i_{s,t}}^{\alpha_{H,s}} K_{G,t}^{\gamma_{G,s}}, \tag{6}$$

where $N^i_{s,t}$, $K^i_{s,t}$, and $H^i_{s,t}$ denote the labor, private capital, and intermediate inputs used by producer i in sector s, while $\alpha_{N,s}$ and $\alpha_{H,s}$ are the value-added labor share and gross-output intermediate-input share, respectively.

As in Baxter and King (1993), Leeper et al. (2010b), and Ramey (2021), the stock of public capital, $K_{G,t}$, affects the production of private goods. The heterogeneous sectoral elasticities $\gamma_{G,s}$ discipline the extent to which public capital enhances the productivity of the gross output of sector s.

Each producer i sells its sectoral variety to the wholesalers at price $P^i_{s,t}$. Prices maximize profits and are subject to a Calvo (1983) price-setting friction, such that producers reset their price with the sector-specific probability $1 - \phi_s$.

2.2.2 Wholesalers

In each sector, the wholesalers buy the different varieties of the sectoral good to produce the final sectoral good $Z_{s,t}$ using the CES technology

$$Z_{s,t} = \left(\int_0^1 {Z^i_{s,t}}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{7}$$

with elasticity of substitution across within-sector varieties ϵ. The price of the sectoral good, $P_{s,t}$, is

$$P_{s,t} = \left(\int_0^1 {P_{s,t}^{i}}^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \tag{8}$$

The goods are sold to retailers, which produce the bundles used in the production of private and public consumption, private and public investment, and intermediate inputs. Accordingly, the sectoral resource constraint reads

$$Z_{s,t} = C_{s,t} + I_{s,t} + \sum_{x=1}^{S} H_{x,s,t} + G_{s,t} + I_{G,s,t}, \tag{9}$$

where $C_{s,t}$ is the demand of private-consumption retailers of sector-s goods, $I_{s,t}$ is the demand of private-investment retailers, $H_{x,s,t}$ is the demand of intermediate-input retailers associated to sector x, $G_{s,t}$ is the demand of public-consumption retailers, and $I_{G,s,t}$ is the demand of public-investment retailers.

2.2.3 Retailers

The public-investment retailers purchase the goods $I_{G,s,t}$ at price $P_{s,t}$, and produce the aggregate public investment good using the CES technology

$$I_{G,t} = \left[\sum_{s=1}^{S} \omega_{I_G,s}^{\frac{1}{\nu_{I_G}}} I_{G,s,t}^{\frac{\nu_{I_G}-1}{\nu_{I_G}}} \right]^{\frac{\nu_{I_G}}{\nu_{I_G}-1}}, \tag{10}$$

where $\omega_{I_G,s}$ is a sectoral weight and ν_{I_G} denotes the elasticity of substitution of public-investment goods across sectors. The final public-investment good is sold to the fiscal authority at price $P_{I_G,t}$:

$$P_{I_G,t} = \left[\sum_{s=1}^{S} \omega_{I_G,s} P_{s,t}^{1-\nu_{I_G}} \right]^{\frac{1}{1-\nu_{I_G}}}. \tag{11}$$

A similar structure applies for the bundles of private consumption (with weights $\omega_{C,s}$ and elasticity of substitution ν_C), public consumption (with weights $\omega_{G,s}$ and elasticity of substitution ν_G), private investment (with weights $\omega_{I,s}$ and elasticity of substitution ν_I), and sectoral intermediate inputs (with weights $\omega_{H,s,x}$

and elasticity of substitution ν_H). We report all the details in Appendix A.

2.3 Government

The government consists of a monetary and fiscal authority. The monetary authority sets the nominal interest rate subject to a standard Taylor rule

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\phi_\rho} \left(\pi_t^{\phi_\pi} X_t^{\phi_x}\right)^{1-\phi_\rho}, \tag{12}$$

where $\bar{R}$ is the steady-state value of the nominal interest rate,[7] π_t is the aggregate inflation rate defined over the GDP deflator, and $X_t = Y_t / Y_t^{\text{flex}}$ is the output gap, defined as the ratio between real GDP, Y_t, and its corresponding value in a flexible-price economy, Y_t^{flex}. The parameter ϕ_ρ denotes the degree of interest-rate inertia. The parameters ϕ_π and ϕ_x denote the responsiveness of the nominal interest rate to aggregate inflation and the output gap, respectively.

The fiscal authority sets the lump-sum nominal tax on households, T_t, to finance exogenous streams of public consumption, G_t, and public investment, $I_{G,t}$.[8] The government purchases the public-consumption and investment goods from the two associated retailers at prices $P_{G,t}$ and $P_{I_G,t}$, respectively.

Public-consumption good purchases, G_t, and planned public-investment ex-

[7] Throughout the paper, we denote the steady-state value of a given variable A_t as $\bar{A}$.

[8] In the data, the largest component of government consumption spending is the compensation of public employees (see Moro and Rachedi, 2022). We abstract from it following the vast literature that treats public spending solely as the purchases of goods from private industries. Appendix F evaluates the robustness of our results to the case in which public consumption expenditures also consist of the compensation of public employees.

penditures, $\tilde{I}_{G,t}$, are determined exogenously by the autoregressive processes

$$\log G_t = (1-\rho)\log \bar{G} + \rho \log G_{t-1} + \epsilon_{G,t}, \tag{13}$$

$$\log \tilde{I}_{G,t} = (1-\rho)\log \bar{I}_G + \rho \log \tilde{I}_{G,t-1} + \epsilon_{I,t}, \tag{14}$$

where ρ denotes the persistence of the processes, $\bar{G}$ and $\bar{I}_G$ are the steady-state values of public consumption and investment, $\epsilon_{G,t}$ is the public-consumption shock, and $\epsilon_{I,t}$ is the public-investment shock.

As in Leeper et al. (2010b) and Ramey (2021), public investment features time-to-spend and time-to-build frictions. The time-to-spend constraint implies that planned public investment expenditures lead actual spending with a lag:

$$I_{G,t} = \frac{1}{\zeta}\sum_{j=1}^{\zeta} \tilde{I}_{G,t-j-1}. \tag{15}$$

Accordingly, current public investment averages lagged planned expenditures, with ζ capturing the horizon of the delay. The time-to-build friction implies that actual public investment accumulates into public capital with a lag μ:

$$K_{G,t} = (1-\delta_{K_G})\, K_{G,t-1} + I_{G,t-\mu}, \tag{16}$$

where δ_{K_G} denotes the depreciation rate of public capital.

In every period, taxes equal spending to balance the government budget

$$T_t = P_{G,t}G_t + P_{I_G,t}I_{G,t}. \tag{17}$$

2.4 Closing the Model

Summing across the nominal sectoral value added $\mathcal{Y}_{s,t}$—which equals the difference between the nominal values of sectoral gross output and sectoral intermediate inputs—yields the nominal GDP of the economy, $\mathcal{Y}_t$. In turn, $\mathcal{Y}_t$ equals the nominal values of private and public demand for consumption and investment:

$$\mathcal{Y}_t = \sum_{s=1}^{S} \mathcal{Y}_{s,t} = P_{C,t}C_t + P_{I,t}I_t + P_{G,t}G_t + P_{I_G,t}I_{G,t}. \tag{18}$$

Finally, real aggregate GDP is defined as the ratio between nominal aggregate GDP and the GDP deflator,[9] P_t, that is,

$$Y_t = \mathcal{Y}_t / P_t. \tag{19}$$

3 Calibration

The model is calibrated to the U.S. economy at quarterly frequency. The economy features 55 sectors, reflecting the 3-digit disaggregation level of NAICS codes.[10] We report here the entire calibration strategy, and refer to Appendix B for the list of the sectors and further details.

We consider a zero inflation rate in the steady state. The household's discount rate $\beta = 0.995$ targets a 2% real annual steady-state interest rate. The risk aversion coefficient is $\sigma = 2$, and we set $\eta = 0.67$ to imply a Frisch elasticity of 1.5.[11]

To set the (short-run) elasticity of substitution of private consumption, private

[9] The GDP deflator is defined as the ratio between nominal value added and the value added measured with steady-state prices.

[10] This is the maximum level of disaggregation necessary for deriving the series of sectoral utilization-adjusted TFP, which is required to estimate the sector-specific output elasticity to public capital.

[11] This value is higher than the estimates of the Frisch elasticity at the individual level (Chetty et al., 2013). However, Erosa et al. (2016) show that a low individual Frisch elasticity is consistent with an aggregate labor supply elasticity of 1.75. We decide to set this relatively high value of the Frisch elasticity since it helps the model in generating fiscal multipliers in line with the empirical evidence (Hall, 2009). For instance, Baxter and King (1993) and

investment, and intermediate inputs across sectors, we rely on empirical findings that uncover these parameters at business cycle frequencies. Specifically, the elasticity of substitution of private consumption across sectors is $\nu_C = 2$, in line with the estimate of Hobijn and Nechio (2019). Similarly, we set the elasticity of substitution of private investment across sectors to $\nu_I = 2$. The elasticity of substitution of intermediate inputs across sectors is $\nu_H = 0.1$, following the evidence of Atalay (2017) and Boehm et al. (2019).

As in Bouakez et al. (2023), we set the elasticity of substitution of public consumption across sectors to $\nu_G = 1$. This choice ensures that the sectoral composition of public spending is kept constant over time. Likewise, the elasticity of substitution of public investment to $\nu_{I_G} = 1$.

Given the elasticities of substitution, we discipline $\omega_{C,s}$, $\omega_{G,s}$, $\omega_{I,s}$, $\omega_{I_G,s}$, and $\omega_{H,s,x}$ with the 2019 U.S. Bureau of Economic Analysis (BEA) Input-Output Tables by targeting, respectively: the sectoral shares in personal consumption expenditures, government consumption spending, nonresidential private fixed investment, nonresidential public fixed investment, and the supply and use of intermediate inputs with respect to all industries.

The elasticity of substitution across within-sector varieties is $\epsilon = 4$ to match the 25% markup estimated in De Loecker et al. (2020). Given markups, the value-added labor shares, $\alpha_{N,s}$, target the sectoral shares of employee compensation in value added. The gross-output intermediate-input shares, $\alpha_{H,s}$, target the sectoral shares of intermediate-input expenditures in gross output (net of taxes on production and imports, and subsidies). The next subsection details the estimation of the elasticity of sectoral output to public capital, $\gamma_{G,s}$.

Ramey (2021) consider a Frisch elasticity of 4. Table F.1 in Appendix F shows that our main results hold also in a version of the model with a Frisch elasticity of 1.

We set the sectoral price rigidity, ϕ_s, using the duration of prices provided by Nakamura and Steinsson (2008). In respect to the monetary rule, we choose the parameters $\phi_\rho = 0.8$, $\phi_\pi = 1.5$, and $\phi_x = 0.2$, following the evidence of Clarida et al. (2000). Regarding the fiscal authority, we set the persistence of the public-consumption and public-investment shocks to $\rho = 0.95$, as in Leeper et al. (2010b) and Ramey (2021). The steady-state values for public consumption, $\bar{G}$, and public investment, $\bar{I}_G$, match the shares of nominal government consumption expenditures (14%) and nominal government gross investment (3.5%), as fractions of nominal GDP in 2019. We then follow Leeper et al. (2010b) to set the time-to-build horizon for public capital to $\mu = 4$, and the time-to-spend horizon to $\zeta = 3$.

The elasticity of labor across sectors is set to $\nu_N = 1$ following the estimate of Horvath (2000). Similarly, the elasticity of capital across sectors is $\nu_K = 1$. We calibrate the parameter that governs the private-investment adjustment cost, $\Omega = 7.25$, to match the relative volatility of investment to output obtained from HP-filtered ratio of real nonresidential investment with respect to real GDP from 1950Q1 to 2019Q4 in a model version featuring only aggregate TFP shocks. The depreciation rates of private and public capital are $\delta_K = 0.015$ and $\delta_{K_G} = 0.01$, respectively, based on the estimates of Ramey (2021).

3.1 The Sectoral Output Elasticity of Public Capital

In the model, public capital raises firms' productivity. To capture the idea that some industry may benefit more than others, the output elasticity to public capital, $\gamma_{G,s}$, varies across sectors. Following Bouakez et al. (2017) and Ramey (2021), we discipline this dimension by estimating a cointegrating relationship between TFP and the stock of public capital. In doing so, we extend their time-series methodology to a panel setting. Instead of regressing the logarithm of *aggregate* TFP on the logarithm of the aggregate stock of public capital, our dependent variable is the logarithm of *sectoral* TFP:

$$\log TFP_{s,t} = \gamma_{G,s} \log K_{G,t} + \epsilon_{s,t}. \tag{20}$$

We then estimate the elasticity of sectoral gross output to public capital using the heterogeneous cointegrated panel approach of Pedroni (2001).[12]

To correctly identify the output elasticity, Bouakez et al. (2017) and Ramey (2021) argue that the TFP measure should be adjusted to account for the variable utilization of production factors. While a utilization-adjusted TFP series has been constructed at the aggregate level (Fernald, 2014), there are no sufficiently long time-series to estimate a cointegrating relationship at the sectoral level. Following Basu et al. (2006) and Fernald (2014), we build utilization-adjusted sectoral TFP series for the 55 industries of our model. To do so, we use KLEMS data from 1963 to 2016 on: real and nominal gross output, real and nominal intermediate inputs, the stock of five types of capital (IT, software, R&D, art, and other) and the use of college and non-college labor.[13] We complement this information jointly with the chain-type quantity index for the net stock of total government fixed assets, provided by the Fixed Assets Accounts Tables of the BEA.

We estimate an average output elasticity of public capital of 0.0575. This value is in line with the elasticity of 0.05 used in Baxter and King (1993), Leeper

[12] Unlike Bouakez et al. (2017), we do not control for the stock of education and R&D expenditures. If we do so, the average output elasticity to public capital shrinks to 0.033, well below the range of estimates derived in the literature. While Ramey (2021) thoroughly documents that these controls reduce the upward bias in the estimates of the elasticity due to the positive correlation of the error of Equation (20) with the stock of public capital, she also conjectures that these additional controls may lead to a downward estimation bias.

[13] Appendix C describes how we derived the sector-specific utilization-adjusted TFP series.

et al. (2010b), and Ramey (2021), and just slightly lower than the value of 0.065 estimated by Bouakez et al. (2017).[14] Overall, our estimate is conservative and falls at the lower end of the range of elasticities found in the literature.

The novelty of our approach is that we estimate how the elasticity of *sectoral* output to public capital varies across industries. In fact, the average value of 0.0575 conceals a large amount of heterogeneity: the elasticity varies from 0 (the point estimate is slightly negative but not statistically significant) for the pipeline transportation industry up to 0.1363 for the computer manufacturing sector.[15]

4 Aggregate Implications of Public Investment

This section studies the response of aggregate value added to public-investment shocks. The presence of intermediate inputs substantially amplifies the aggregate output response when compared to the average one-sector economy without intermediate inputs.[16] To carry out this exercise, we define the public-investment and public-consumption multipliers in present-value terms. Specifically, the public-investment multiplier at horizon $\mathcal{H}$, $\mathcal{M}_{\mathcal{H}}^{\mathcal{I}_{\mathcal{G}}}$, equals the ratio between the discounted sum of the deviations from steady state of aggregate GDP and the discounted

[14]For context, the meta-analysis of Bom and Ligthart (2014) shows that the average output elasticity to public capital used in the literature ranges between 0.08 and 0.12.

[15]Figure C.1 in Appendix C reports the estimates of the sector-specific elasticities.

[16]The average one-sector economy without intermediate inputs is a model version in which: there are no intermediate inputs, $\alpha_{H,s} = 0$; the value-added labor intensities are set symmetrically across industries to the value-added labor share of the entire economy, $\alpha_{N,s} = \alpha_N$; the output elasticity to public capital and the Calvo price-adjustment frequency are set to their average values across sectors, $\gamma_{G,s} = \gamma_G$ and $\phi_s = \phi$; the contributions to private and public demand are symmetric across sectors, $\omega_{C,s} = \omega_{I,s} = \omega_{G,s} = \omega_{I_G,s} = 1/55$.

sum of the deviations from steady state of public-investment expenditures, both computed up to quarter $\mathcal{H}$ after the realization of the shock:

$$\mathcal{M}_{\mathcal{H}}^{\mathcal{I}_{\mathcal{G}}} = \frac{\sum_{j=0}^{\mathcal{H}} \beta^j \left(P_j Y_j - \bar{P}\bar{Y}\right)}{\sum_{j=0}^{\mathcal{H}} \beta^j \left(P_{I_G,j} I_{G,j} - \bar{P}_{I_G} \bar{I}_G\right)}. \tag{21}$$

Consequently, the multiplier computes the dollar change in (the present discounted value of) aggregate output associated with a one-dollar rise in the (present discounted) value of public investment. Analogously, the public-consumption multiplier at horizon $\mathcal{H}$, $\mathcal{M}_{\mathcal{H}}^{\mathcal{G}}$, is

$$\mathcal{M}_{\mathcal{H}}^{\mathcal{G}} = \frac{\sum_{j=0}^{\mathcal{H}} \beta^j \left(P_j Y_j - \bar{P}\bar{Y}\right)}{\sum_{j=0}^{\mathcal{H}} \beta^j \left(P_{G,j} G_j - \bar{P}_G \bar{G}\right)}. \tag{22}$$

Our baseline analysis focuses on the long-run multipliers, where $\mathcal{H} \to \infty$.[17] Section 4.2 also evaluates the short-run propagation of public spending by computing the multipliers at any horizon $\mathcal{H}$ between 1 and 60 quarters (15 years).

4.1 Amplification of the Aggregate Output Multiplier

How does sectoral heterogeneity and the presence of intermediate inputs alter the aggregate effects of public investment? To answer this question, Panel A of Table 1 compares the long-run public-investment multipliers implied by our baseline production-network economy and the average one-sector economy. In the average one-sector economy, the long-run public-investment multiplier equals 1.27. This value is at the lower end of the model estimates provided in the literature. This is partly due to our conservative choice of 1.5 for the Frisch elasticity.

[17] We compute the long-run multipliers using the first 2,000 realizations of the variables' responses to the shocks. For public consumption, 90% of the cumulative response of aggregate value added occurs within 39 quarters, while it extends to 268 quarters for public investment. This is consistent with the evidence of Antolin-Diaz and Surico (2022), in which the multiplier does not fully converge even after 15 years.

Table 1: Long-Run Public-Investment and Public-Consumption Multipliers.

One-Sector Economy	Production-Network Economy	Δ%	Δ$
(1)	(2)	(3)	(4)
Panel A: Long-Run Public-Investment Multipliers, $\mathcal{M}_{\infty}^{\mathcal{I}_{\mathcal{G}}}$			
1.27	2.12	68%	0.86
Panel B: Long-Run Public-Consumption Multipliers, $\mathcal{M}_{\infty}^{\mathcal{G}}$			
0.35	0.45	30%	0.10

Note: Panel A reports the long-run public-investment multipliers in the average one-sector economy in Column (1), the baseline production-network economy in Column (2), as well as the difference in the multipliers between the production-network economy and the average one-sector economy in percentage values and absolute values in Columns (3) and (4), respectively. Panel B reports similar statistics for the public-consumption multipliers.

For instance, Ramey (2021) finds a multiplier of 1.7 with a Frisch elasticity of 4. In our production-network economy, the public-investment multiplier is substantially larger, 2.12. Moving from the one-sector to the production-network economy yields an amplification of the public-investment multiplier of 68%. The amplification is also economically significant, as one dollar of public investment yields an additional 86 cents of aggregate value added.[18]

Panel B of Table 1 explores how the amplification of the public-investment multiplier compares with the one of the public-consumption multiplier. Moving from the one-sector to the production-network economy raises the public-consumption multiplier by 30% (and 10 cents). While the amplification is significant and in line with the findings in Bouakez et al. (2023), it is not as large as the one observed for public investment. This comparison highlights the first contribution of our paper: the production-network amplification of the public-investment multiplier is twice as large as that of the public-consumption multiplier.

Appendix F shows that the amplification of the public-investment multiplier holds in an extensive battery of robustness checks, which allow for distortionary taxes (Leeper et al., 2010a), sticky wages (Erceg et al., 2000), an investment network (Vom Lehn and Winberry, 2022), and durable consumption (Boehm, 2020). By and large, the amplification of the public-investment multiplier in the production network economy relative to the one-sector model is consistently around 60%-80%, and at least twice as large as the one of the public-consumption multiplier.

4.2 Multipliers Amplification in the Short Run

The production-network amplification remains substantial even at shorter horizons. This finding is particularly interesting, since it is well documented that

[18]Appendix D confirms that these findings hold through when looking separately at private consumption and investment.

public consumption has a relatively more pronounced effect on aggregate output in the short run (Boehm, 2020; Ramey, 2021).

To illustrate this point, Figure 1 displays the multipliers from Equations (21 and (22) for all quarters up to 15 years (i.e., $\mathcal{M}_{\mathcal{H}}^{\mathcal{I}_{\mathcal{G}}}$ and $\mathcal{M}_{\mathcal{H}}^{\mathcal{G}}$ with $\mathcal{H} \in \{1, \ldots, 60\}$). Consistent with Boehm (2020) and Ramey (2021), the impact effect of public investment on GDP is lower than the one triggered by public consumption, due to the time-to-build and time-to-spend delays, as well as differential crowding out effects on private demand caused by the two fiscal instruments.

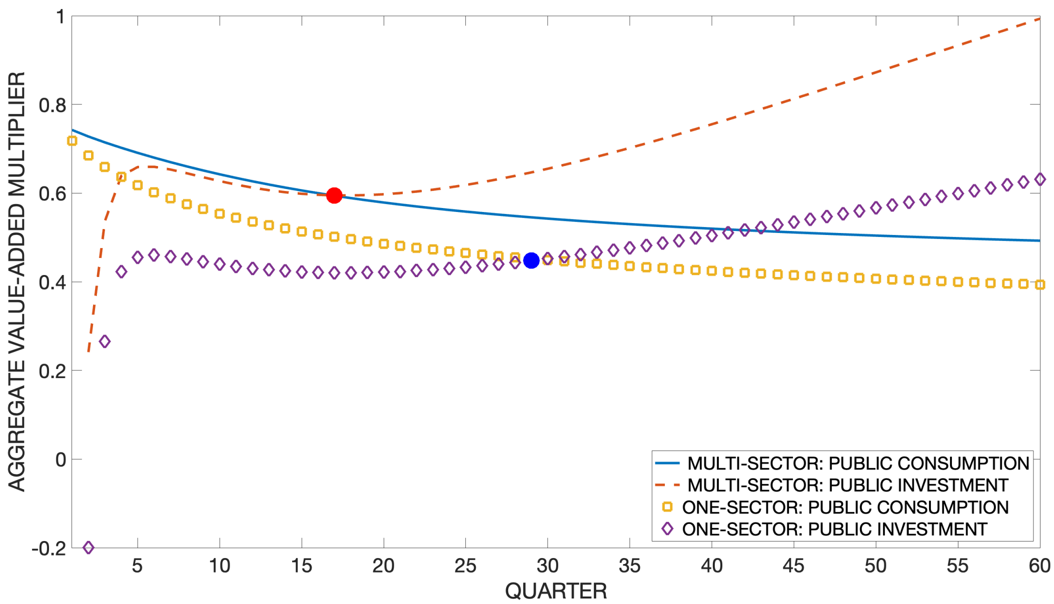
In the average one-sector model, the public-investment multiplier on impact is negative (-0.2), while the public-consumption multiplier is positive (0.72). The public-consumption multiplier then gradually decreases with the horizon $\mathcal{H}$, whereas the opposite applies to the public-investment multiplier. After seven years (28 quarters), the output effect of public investment exceeds that spurred by public consumption.

In the multi-sector economy, the public-investment multiplier in the production network is always positive, also on impact. Although it takes 4 years (17 quarters) for the public-investment multiplier to surpass the public-consumption multiplier, the two multipliers are virtually identical after *just* 6 quarters. This results cannot be overstated: the production network substantially front-loads the aggregate effects of public investment and dramatically reduces the lag required for public investment to be as stimulative as public consumption.[19]

All in all, sectoral inter-linkages allow the model to be consistent with the evidence showing that the stimulus effects of public investment are limited on impact (Boehm, 2020) but become significant after two years (Ilzetzki et al., 2013).

[19] Appendix G provides additional results on the short-run amplification of public investment through the production network.

Figure 1: Dynamics of Public-Consumption and Public-Investment Multipliers.



Note: The blue solid line and the dash orange line represent, respectively, the public-consumption and investment-multiplier in the production network computed at any horizon between 1 and 60 quarters. Analogously, the yellow squares and the violet diamonds represent, respectively, the public-consumption and public-investment multiplier in the average one-sector economy.

4.3 Inspecting the Amplification Mechanism

What drives the amplification of the long-run public-investment multiplier in the production network and to what extent it differs from that of public consumption? To address this question, we examine six alternative multi-sector model specifications which abstract in turn from: (i) the Input-Output matrix, $\alpha_{H,s} = 0$; (ii) heterogeneity in inter-sectoral linkages, $\omega_{H,s,x} = 1/55$; (iii) sectoral heterogeneity in price rigidity, by setting the Calvo parameters to their average value, $\phi_s = \phi$; (iv) heterogeneity in sectors' contribution to public demand, $\omega_{G,s} = \omega_{I_G,s} = 1/55$; ($v$) heterogeneity in sectors' contribution to final demand, $\omega_{C,s} = \omega_{I,s} = \omega_{G,s} = \omega_{I_G,s} = 1/55$; and ($vi$) heterogeneity in factor intensities, by setting them to their economy-wide values, $\alpha_{H,s} = \alpha_H$, $\alpha_{N,s} = \alpha_N$.

Table 2 reports the contribution of each modeling feature to the amplification of both the long-run public-investment and public-consumption multipliers. To make the comparison meaningful, each column shuts down in isolation a different modeling feature *without* altering the implications of the associated average one-sector economy. Accordingly, modeling features whose absence implies lower multipliers are the key ones to the amplification mechanism.

The public-investment amplification is entirely due to the presence of intermediate inputs. If we consider a multi-sector model with heterogeneity in all dimensions but with no production network, the public-investment multiplier drops from 2.12 to 1.24, which almost exactly replicates the 1.27 multiplier of the average one-sector economy. The other five model dimensions barely matter.

In contrast, the amplification of the public-consumption multiplier is due to both the presence of the Input-Output matrix and heterogeneity in the price rigidity across sectors, confirming the findings in Bouakez et al. (2023). Both modeling features flatten the aggregate Phillips curve, thus triggering a mildly inflationary rise in GDP, which does not require a spike in nominal interest rates.

Table 2: Sources of Amplification of the Aggregate Value-Added Multiplier.

Alternative Production-Network Economies Without ...					
IO Matrix	IO Matrix Heterogeneity	Price Rigidity Heterogeneity	Public Demand Heterogeneity	Final Demand Heterogeneity	Factor Intensities Heterogeneity
(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Long-Run Public-Investment Multipliers, $\mathcal{M}_{\infty}^{\mathcal{I}_{\mathcal{G}}}$					
1.24	2.11	2.04	2.29	2.30	2.17
Panel B: Long-Run Public-Consumption Multipliers, $\mathcal{M}_{\infty}^{\mathcal{G}}$					
0.40	0.43	0.41	0.49	0.44	0.44

Note: Panel A reports the long-run public-investment multipliers in six alternative specifications of the production-network economy: without Input-Output matrix, $\alpha_{H,s} = 0$, Column (1); without heterogeneity in the Input-Output matrix, $\omega_{H,s,x} = 1/55$, Column (2); without heterogeneity in price rigidity, $\phi_s = \phi$, Column (3); without heterogeneity in the contributions to public demand, $\omega_{G,s} = \omega_{I_G,s} = 1/55$, Column (4); without heterogeneity in the contributions to final demand, $\omega_{C,s} = \omega_{I,s} = \omega_{G,s} = \omega_{I_G,s} = 1/55$, Column (5); without heterogeneity in the factor intensities, $\alpha_{N,s} = \alpha_N$ and $\alpha_{H,s} = \alpha_H$, Column (6). Panel B reports similar statistics for the long-run public-consumption multiplier.

Table 3 shows that the amplification of the public-investment multiplier crucially hinges on the *level* of the output elasticity to public capital, and not its *heterogeneity*. To establish this result, we measure the public-investment multiplier in four model specifications that differ only in the calibration of the output elasticity to public capital. Since changes in $\gamma_{G,s}$ do not alter the public-consumption multiplier, we focus solely on the public-investment multiplier.

We start by comparing the baseline model (Panel A) to an economy where the output elasticity to public capital is zero, $\gamma_{G,s} = 0$ (Panel B). In this case, public investment resembles public consumption, with the additional time-to-build and time-to-spend delays. The amplification is minimal: the public-investment multipliers of the one-sector and production-network economy equal to 0.32 and 0.37, respectively. Thus, absent a productive public capital, inter-sectoral linkages barely matter for the propagation of public investment.

Panel C shows that *heterogeneity* in the elasticity of public capital across sectors is not quantitatively relevant. When we focus on an economy in which the elasticity is homogeneous across sectors and equal to the average value, the public-investment multiplier barely changes, from 2.12 to 2.05. The reason is that heterogeneity in the sectoral elasticity does not correlate with sectors' position in the production network. If we assign the highest elasticities to the most upstream sectors (inducing a correlation of 1 between the public-capital elasticity and sector's centrality), the public-investment multiplier rises up to 2.46 (Panel D). Instead, if we do the opposite, the multiplier drops by 30%, down to 1.72 (Panel E). Intuitively, a higher elasticity in upstream sectors allows these industries to benefit relatively more from the expansion in public investment. Since these sectors provide intermediate inputs to all industries, the positive effects of public capital propagate through the Input-Output matrix, ultimately raising

Table 3: Public-Investment Multipliers and the Output Elasticity to Public Capital.

	One-Sector Economy	Production-Network Economy	Δ%	Δ$
	(1)	(2)	(3)	(4)
Panel A: Baseline Elasticities to Public Capital				
	1.27	2.12	68%	0.86
Panel B: No Output Elasticity to Public Capital				
	0.32	0.37	18%	0.06
Panel C: No Heterogeneity in the Output Elasticity to Public Capital				
	1.27	2.05	62%	0.79
Panel D: Higher Output Elasticity to Public Capital in Upstream Sectors				
	1.27	2.46	94%	1.19
Panel E: Higher Output Elasticity to Public Capital in Downstream Sectors				
	1.27	1.72	36%	0.46

Note: This table reports similar statistics on the long-run public-investment multiplier to Table 1 with the difference that the sectoral output elasticities to public capital: are set to the baseline values in Panel A; are set to zero in Panel B; are set to average value across sectors in Panel C; are sorted such that the largest values are assigned to the most upstream sectors in Panel D; are sorted such that the largest values are assigned to the most downstream sectors in Panel E.

the production of all sectors.[20]

4.4 Analytical Intuition

This section analytically formalizes the mechanism through which the output response to public investment crucially depends on the presence of intermediate inputs and the value of the public-capital elasticity.

Consider a simplified version of our model, with one sector ($S = 1$), no physical capital ($\alpha_N = 1$), flexible prices ($\phi = 0$), and perfectly competitive goods markets ($\epsilon \to \infty$). Let us abstract from public consumption ($G_t = 0$), and set public investment to a fraction χ of aggregate GDP ($I_{G,t} = \chi Y_t$). Let us assume full depreciation of public capital ($\delta_{K,G} = 1$), no time-to-build and time-to-spend delays ($\zeta = 0$ and $\mu = 0$). In addition, consider a utility function that is logarithmic over consumption ($\sigma = 1$) and linear over labor ($\eta = 0$). With these restrictions, the model is static and can be solved analytically. The households' problem reduces to

$$\max_{C_t, N_t} \log C_t - \theta N_t, \qquad \text{s.t. } W_t N_t = C_t + T_t, \tag{23}$$

the government budget constraint reads

$$I_{G,t} = T_t, \tag{24}$$

and the gross-output production function equals

$$Z_t = N_t^{1-\alpha_H} H_t^{\alpha_H} K_{G,t}^{\gamma_G}. \tag{25}$$

We then solve for output and evaluate how it changes relative to its equilib-

[20]This result resembles the fact that distortions and industrial policy interventions upon upstream sectors have the largest aggregate effects (Liu, 2019; Bigio and La'O, 2020; Buera and Trachter, 2023).

rium level after a surge in public investment, modeled as an increase of χ. Our simplified model implies that the response of aggregate value added to public investment increases with the share of intermediate inputs, α_H,

$$\frac{\partial^2 Y_t}{\partial \chi \partial \alpha_H} / Y_t = \frac{\gamma_G}{(1-\chi)\,\chi\left[\gamma_G - (1-\alpha_H)\right]^2} > 0. \tag{26}$$

Importantly, the derivative is positive only as long as public capital is productive. When $\gamma_G = 0$, the share of intermediate inputs is immaterial for the response of output to public investment. In other words, the stimulus effect of public investment crucially depends on the interaction between the presence of intermediate inputs and the output elasticity to public capital.[21]

This analysis also illustrates the differential amplification channels of public investment and public consumption. In our simple model, the output response to public consumption does not vary with the share of intermediate inputs since public consumption is not productive. Bouakez et al. (2023) overturn this irrelevance result through sticky prices. In this case, intermediate inputs boost the output response to public consumption by flattening the aggregate Phillips curve.

How does the presence of a production network amplify the public-investment multiplier? The gross-output technology in (25) implies that value added equals

$$Y_t = (1-\alpha_H)\,\alpha_H^{\frac{\alpha_H}{1-\alpha_H}} N_t K_{G,t}^{\frac{\gamma_G}{1-\alpha_H}}. \tag{27}$$

This formulation crystallizes that any given *gross-output elasticity* to public capital, γ_G, implies a relatively higher *value-added elasticity* to public capital, $\gamma_G/(1-\alpha_H)$, which increases with the share of intermediate inputs in gross out-

[21]Appendix H extends this simplified model to two sectors to establish that the aggregate effect of public investment is magnified when the elasticity to public capital is relatively larger in the upstream sector.

put, α_H. Intuitively, intermediate inputs amplify the productivity-enhancing effect of public capital as firms not only benefit directly from a surge in public investment, but do so also indirectly. Specifically, public investment raises the efficiency in the provision of intermediate inputs, and thus curtails their cost. Consequently, firms expand the purchase of intermediate inputs and optimally scale up their production.

This result sheds novel insights on the values of the output elasticity to public capital that should be used to discipline one-sector models. Estimates that recover the elasticity from a technology in gross-output terms should be mapped into one-sector economies by adjusting for the intermediate-input share, as illustrated in Equation (27). Consistently with this logic, our production-network with an average elasticity of 0.0575 yields a multiplier that can be reproduced by a one-sector economy if the elasticity is set to $0.0575/(1-0.4779) = 0.1105$, where 0.4779 is the economy-wide share of intermediate inputs in gross output.

4.5 The Socially Optimal Amount of Public Capital

The amplification of the multiplier also depends on the way in which the production network alters the optimal level of public capital.[22] Ramey (2021) shows a stronger output response to public-investment shocks when public capital is below its optimal level. In our simplified model, intermediate inputs raise the optimal amount of public capital for any given output elasticity to public capital. Specifically, the optimal ratio of public capital to GDP equals

$$\frac{\bar{K}_G}{\bar{Y}} = \beta \frac{\gamma_G}{1-\alpha_H}. \tag{28}$$

[22] As in Ramey (2021), we refer to the optimal amount of public capital as that maximizing households' steady-state utility. For the analysis on the optimal public spending in recessions, see Bouakez et al. (2020).

Thus, intermediate inputs lead to a higher socially optimal level of public capital, highlighting once again the key role of the interaction between the public-capital productivity and the production network.

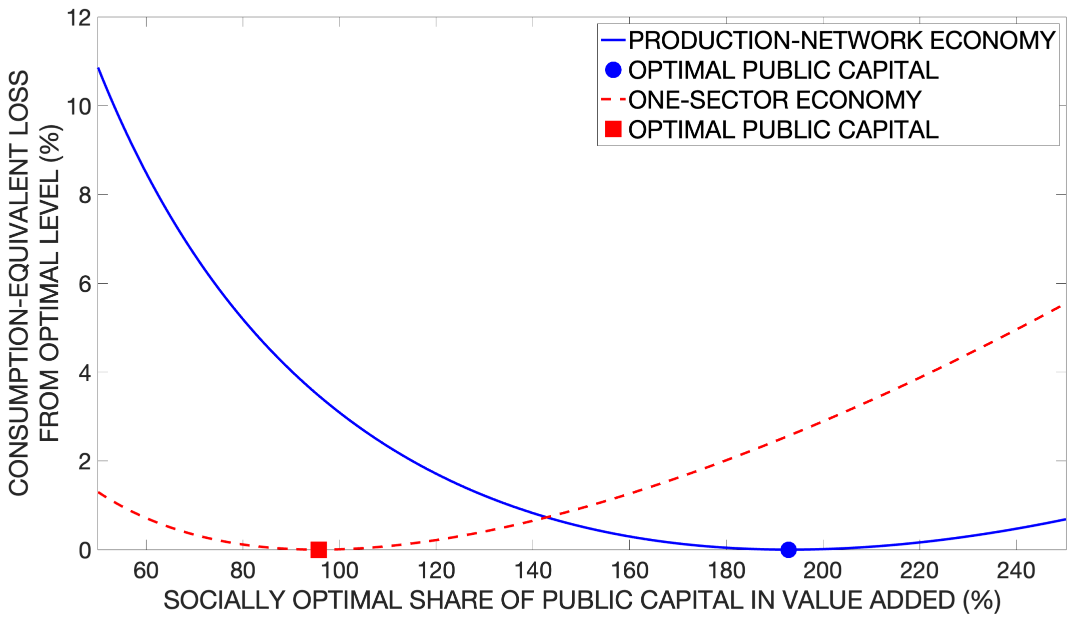
The optimal amount of public capital *doubles* when moving from the one-sector to the production-network economy: the ratio of optimal public capital to annualized aggregate value added equals 98.5% in the former, and 198.7% in the latter. Similarly, the optimal level of public investment changes from 3.9% to 7.8%.[23] To put these numbers in perspective, in the data public capital equals 73% of GDP, whereas public investment is 3.3% of GDP (Ramey, 2021). Thus, accounting for sectoral heterogeneity and inter-linkages leads to an optimal level of public capital which is way above that observed in the data.

Interestingly, our model yields also novel insights on the costs associated to inefficient levels of public capital. Figure 2 reports how the welfare losses (in consumption-equivalence terms) vary with the share of public capital in value added in the production-network and one-sector economy.[24] The production-network economy implies substantially lower welfare costs for inefficient low levels of public capital, and shifts the impact of inefficient low levels when compared to the one-sector economy. In the one-sector economy, the welfare costs of a ratio of public capital to annual GDP of 50% are identical to those associated with a ratio of 166%. Instead, in the multi-sector economy the welfare costs of the former are two order of magnitude larger than those of the latter. In other

[23]Appendix I establishes that the amplification of the optimal level of public capital is entirely due to the presence of intermediate inputs.

[24]Welfare losses in consumption-equivalence terms are computed as the constant rate of change imposed on households' lifetime consumption to bring them to the value they would achieve in an economy with the optimal amount of public capital.

Figure 2: Welfare Loss as a Function of the Share of Public Capital in Value Added.



Note: The blue solid line and the dash orange line represent, respectively, the welfare loss—measured in consumption-equivalent terms—as a function of the share of public capital in value added for the production-network economy and the average one-sector economy.

words, inefficiently low levels of public capital become particularly costly when accounting for inter-sectoral linkages.

5 Sectoral Implications of Public Investment

How is the stimulus effect of public spending allocated across industries? We document that public investment is concentrated in a handful of industries, while public consumption features contributions from nearly all sectors. In spite of this remarkable concentration, the interaction between intermediate inputs and the output elasticity to public capital plays a key role in distributing more evenly the output gains associated with public investment across sectors. We conclude by validating this model prediction in the data.

5.1 Sectoral Concentration of Public Investment

How do government purchases vary across sectors? To address this question, we compare the sectoral contribution to government consumption and investment spending, defined as the share of total spending which is allocated to each individual sector, as derived from the 2019 Input-Output Tables.

We document a sharp concentration in the industrial composition of public investment: only nine sectors contribute more than 0.5% of total public investment. Notably, the lion's share of public investment comes from just three industries: (*i*) construction, (*ii*) professional, scientific and technical services, and (*iii*) computer systems services, with a total joint share of 78%. This marked concentration stands in stark contrast with the sectoral composition of public consumption, where 31 sectors have shares above 0.5%, and reaching a share of 78% requires summing over the 15 largest recipient industries.[25] Importantly, while the standard deviation of the sectoral contributions to public consumption roughly corresponds to that of private consumption, the standard deviation of

[25] Appendix J details the allocation of private and public spending across sectors.

the sectoral contributions to public investment is 80% larger than that of private investment. Once more, this finding corroborates the fact that public investment is relatively more concentrated across sectors than public consumption.

5.2 Sectoral Distribution of the Aggregate Multiplier

How do the sectoral contributions to government expenditures shape the dispersion in sectoral value-added responses to public spending? To assess the sectoral implications of the two types of government spending, we need a measure that controls for the (five-fold) difference in magnitude between the aggregate public-investment and public-consumption multipliers. For this purpose, we compute the sectoral distribution of the public-investment and public-consumption aggregate multipliers as the ratios $\mathcal{M}^{\mathcal{I}_\mathcal{G}}_{s,\infty}/\mathcal{M}^{\mathcal{I}_\mathcal{G}}_{\infty}$ and $\mathcal{M}^{\mathcal{G}}_{s,\infty}/\mathcal{M}^{\mathcal{G}}_{\infty}$ between the long-run sectoral value-added multipliers,

$$\mathcal{M}^{\mathcal{I}_\mathcal{G}}_{s,\infty} = \frac{\sum_{j=0}^{\infty} \beta^j \left(\mathcal{Y}_{j,s} - \bar{\mathcal{Y}}_s\right)}{\sum_{j=0}^{\infty} \beta^j \left(P_{I_G,j} I_{G,j} - \bar{P}_{I_G} \bar{I}_G\right)} \qquad \mathcal{M}^{\mathcal{G}}_{s,\infty} = \frac{\sum_{j=0}^{\infty} \beta^j \left(\mathcal{Y}_{j,s} - \bar{\mathcal{Y}}_s\right)}{\sum_{j=0}^{\infty} \beta^j \left(P_{G,j} G_j - \bar{P}_G \bar{G}\right)},$$

and the long-run aggregate value-added multipliers, $\mathcal{M}^{\mathcal{I}_\mathcal{G}}_{\infty}$ and $\mathcal{M}^{\mathcal{G}}_{\infty}$, as defined in Equation (21) and (22). These ratios measure how one additional dollar of aggregate public-spending multiplier is distributed across sectors.

Figure 3 reports the sectoral distribution of the public-investment and public-consumption multipliers across the 55 sectors of our model. These sectoral decompositions crucially depend on the type of fiscal expenditure. Public consumption benefits more industries like professional services, administrative services, and petroleum manufacturing, each absorbing around 10-15 cents per dollar of the aggregate multiplier. Public investment yields the largest gains for sectors such as professional services and construction, with each of them accounting for about 13 cents per dollar of the aggregate multiplier.

While no industry suffers from an upsurge in public investment, 12 sectors report output losses associated with changes in public consumption. Among these, retail trade is the most affected, experiencing a drop in value added of 5 cents for each dollar of the aggregate public-consumption multiplier.

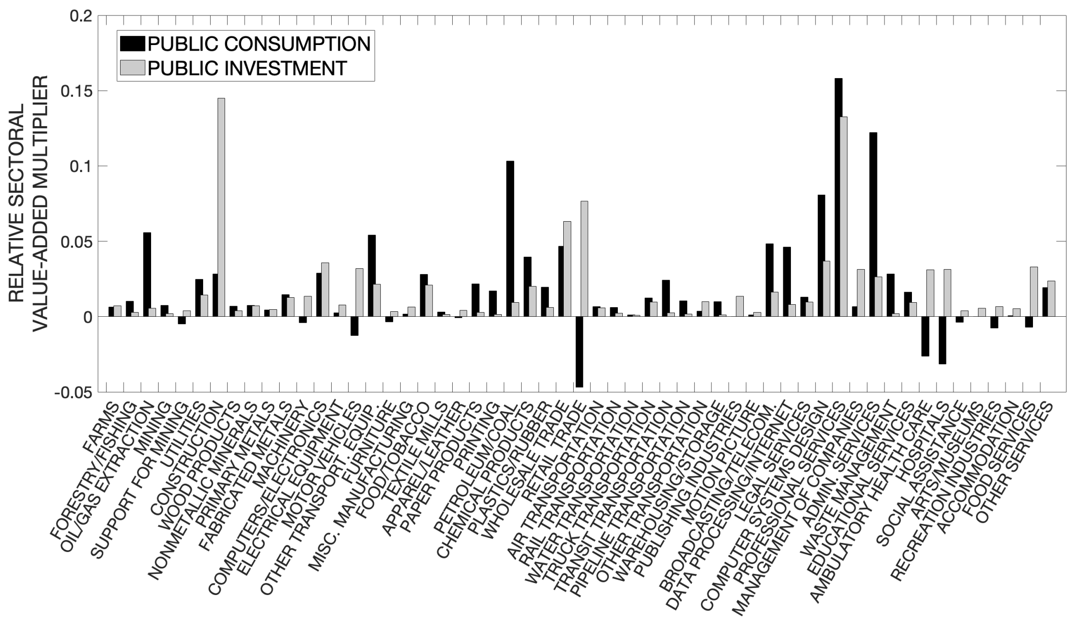
Figure 3 indicates that sectoral responses to public consumption are far more concentrated than those to public investment. To illustrate this point, we count the number of industries that explain 80% of the aggregate spending multiplier: while this share is accounted for only by 11 industries for the case of public consumption, we have to consider up to 19 industries for public investment. This indicates that although the direct contributions to public investment are more concentrated across industries, its output gains are distributed more evenly. To put it in perspective, the standard deviation of sectors' contributions to public consumption and public investment are 2.5% and 7.3%, respectively, as opposed to the standard deviations of the sectoral distribution of the aggregate multipliers, that are 3.5% for public consumption and 2.8% for public investment.

The presence of intermediate inputs is key in yielding a relatively even distribution of the output gains of public investment in spite of the marked concentration in its sectoral contributions. In the aftermath of a public investment shocks, the surge in public capital boosts firms' efficiency. The Input-Output matrix magnifies the positive effects of public capital, as sectors may *indirectly* benefit from the higher efficiency in the provision of intermediate inputs, even when they do not *directly* contribute to the production of public-investment goods. As a result, the output gains propagate to a wider pool of industries. We provide direct evidence on this channel in Table K.1 in Appendix K, by showing that the output gains of public investment become highly concentrated once we abstract from the presence of intermediate inputs.

5.3 Empirical Validation

Our model predicts that sectoral value-added responses to public-investment shocks are relatively more evenly distributed than those generated by public-

Figure 3: The Sectoral Effects of Public Spending.



Note: Sectoral distribution of the aggregate multipliers for both public consumption and public investment in the production-network economy.

consumption shocks. We test this prediction in the data by generalizing the time-series approach of Ramey and Zubairy (2018) to a panel setting. We extend the linear projection method of Jordà (2005) and estimate the following panel regressions for public investment and public consumption, respectively

$$\sum_{t=0}^{\mathcal{H}} \frac{Y_{s,t}}{\tilde{Y}_t} = \beta_{I_G} \sum_{t=0}^{\mathcal{H}} \frac{I_{G,t}}{\tilde{Y}_t} \times \omega_{I_{G,s}} + \alpha_s + \delta_t + \epsilon_{s,t}, \tag{30}$$

$$\sum_{t=0}^{\mathcal{H}} \frac{Y_{s,t}}{\tilde{Y}_t} = \beta_G \sum_{t=0}^{\mathcal{H}} \frac{G_t}{\tilde{Y}_t} \times \omega_{G_s} + \alpha_s + \delta_t + \epsilon_{s,t}. \tag{31}$$

The dependent variable in Equation (30) is the ratio between real sectoral value added and real potential GDP, $\sum_{t=0}^{\mathcal{H}} Y_{s,t}/\tilde{Y}_t$, cumulated up to horizon $\mathcal{H}$. This term is regressed on the interaction between the ratio of real public investment and real potential GDP, also cumulated up to horizon $\mathcal{H}$, and sectors' direct contribution to public investment, $\sum_{t=0}^{\mathcal{H}} \left(I_{G,t}/\tilde{Y}_t\right) \times \omega_{I_{G,s}}$. Equation (31) replaces the independent variable with the interaction between the ratio of cumulated real public consumption and real potential GDP, and sectors' direct contribution to public consumption, $\sum_{t=0}^{\mathcal{H}} \left(G_t/\tilde{Y}_t\right) \times \omega_{G,s}$.

Both regressions include sector fixed effects, α_s, to capture time-invariant unobserved heterogeneity across industries, and time fixed effects, δ_t, to capture the average effect of public spending over sectoral value added. The coefficients β_{I_G} and β_G measure how the value-added multiplier of a given industry varies with its own direct contribution to public spending above and beyond the average sector multiplier. In this way, these estimates naturally map into the way in which the sectoral distribution of the aggregate multipliers varies with sectors' contribution to government expenditures.

We identify these coefficients as in Ramey and Zubairy (2018): (*i*) we focus on the public expenditures of the federal defense government; (*ii*) we uncover the exogenous variation in public spending which is not already incorporated in agents' expectations by instrumenting both types of expenditures with two variables, the military-spending news variable of Ramey (2011), and the timing restriction of Blanchard and Perotti (2002). Both these instruments are interacted with sectors' direct contribution to either public investment or public consumption.

To estimate the coefficients in (30) and (31), we merge information on sectoral value added with data on aggregate public investment and consumption, as well as with sector's direct contribution to public spending. The bulk of our data comes from the BEA. Specifically, we take nominal sectoral value added, nominal public investment from the defense government, and the nominal purchases of goods and services from the defense government as a measure of public consumption. These variables are divided by the associated chain-type price index. We then take the real potential GDP from the estimates of the Congressional Budget Office, and the Ramey (2011)'s news variable from Ramey and Zubairy (2018). We end up with a panel across the 55 sectors of our model at the annual frequency, from 1963 to 2015.

Table 4 reports the estimates of the interaction terms of regressions (30) and (31): the interaction terms for the public-investment and public-consumption regressions are positive and highly statistically significant. Importantly, the estimates confirm the model predictions on how the sectoral distribution of the aggregate multipliers varies with the direct contributions to government spending: the estimates of β_G are larger than those of β_{I_G}, especially at the five-year horizon. These differences are highly economically significant. If we put the estimates in perspective of the variation of the contributions to public investment, one additional standard deviation in the sectoral contributions to fiscal expenditures raises the 5-year sectoral distribution of the public-consumption multiplier by 9 cents, whereas the same increase raises the 5-year sectoral distribution of the public-investment multiplier by less than 5 cents.

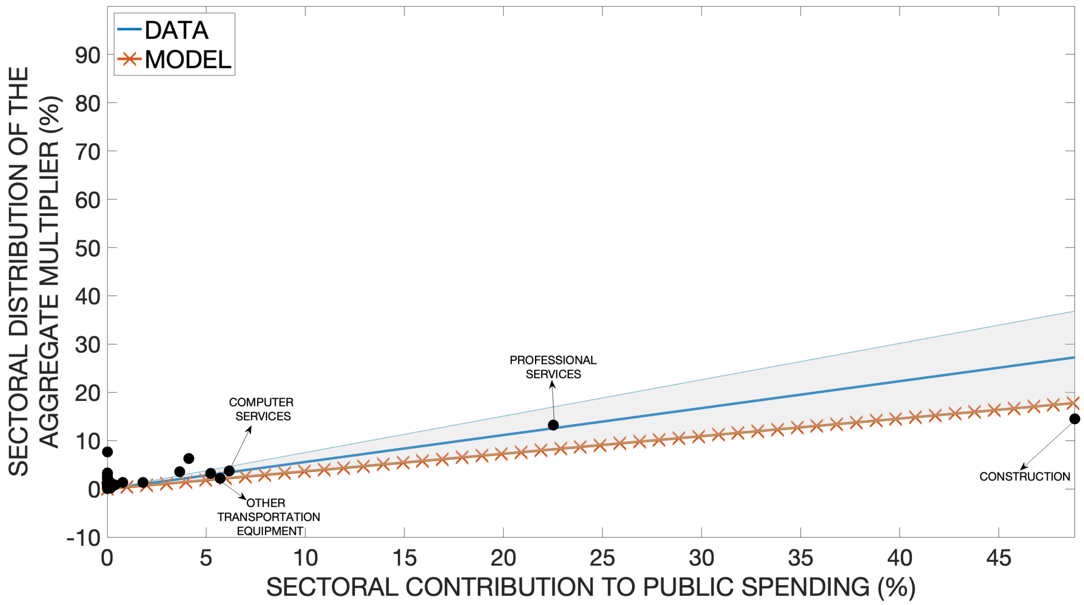
Table 4: Sectoral Implications of Public Consumption and Investment in the Data.

	Dependent Variable: $\sum_{t=0}^{\mathcal{H}} \frac{Y_{s,t}}{\tilde{Y}_t}$			
	$\mathcal{H}=1$		$\mathcal{H}=5$	
	(1)	(2)	(3)	(4)
$\sum_{t=0}^{\mathcal{H}} \frac{I_{G,t}}{\tilde{Y}_t} \times \omega_{I_{G,s}}$	0.66***		0.66***	
	(0.09)		(0.10)	
$\sum_{t=0}^{\mathcal{H}} \frac{G_t}{\tilde{Y}_t} \times \omega_{G_s}$		0.77***		1.21***
		(0.31)		(0.40)
Sector Fixed Effects	YES	YES	YES	YES
Year Fixed Effects	YES	YES	YES	YES
N. Observations	2,805	2,805	2,585	2,585

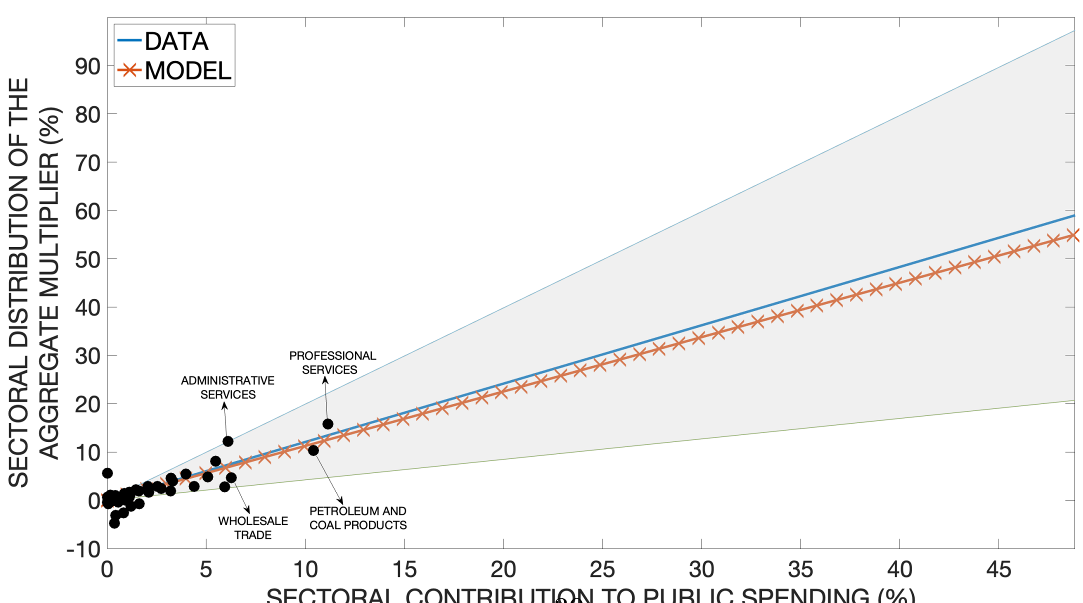
Note: The table reports the estimates of a panel regression across 55 sectors at yearly frequency, from 1963 to 2015. The dependent variable is real sectoral value added scaled by real aggregate potential GDP. Columns (1) and (3) focus on the 1-year and 5-year scaled cumulative defense public-investment shocks, and interact them with sectors' contribution to defense public investment. Columns (2) and (4) focus on the 1-year and 5-year scaled cumulative defense public-consumption shocks, and interact them with sectors' contribution to defense public consumption. Government spending is instrumented with the Blanchard and Perotti (2002) timing restriction and the Ramey (2011) news variable, and their interactions with sectors' contribution to either type of fiscal expenditures. All cases feature industry and year fixed effects. Double-clustered standard errors are reported in parentheses. *** denotes statistical significance at the 1% level.

Figure 4: Sectoral Multipliers and Contribution to Public Spending: Model vs. Data.

Panel (a): Public Investment



Panel (b): Public Consumption



We then compare the estimated sensitivities of the sectoral multipliers to the sectoral direct contributions to public spending with those generated by the model. We do so in Figure 4, which scatters the sectoral distribution of both the public-investment and public-consumption multipliers with respect to sectors' direct contribution to either type of expenditure. Then, we report the regression lines implied by the estimates of β_{I_G} and β_G at the 5-year horizon with those implied by the model.[26]

This exercise highlights two main findings. First, the empirical estimates on how the sectoral distribution of the aggregate multipliers varies with sectors' direct contributions to public spending are remarkably in line with the ones generated by the model. This result lends credence to the quantitative predictions of our production network. Second, the regression lines associated to public consumption are much steeper than those of public investment, so that the sensitivity of the sectoral distribution of the public-consumption multiplier to the sector's direct contribution is relatively higher. For instance, let us take the example of the construction sector, whose contribution to public investment accounts for almost half of its total value. This share implies that this industry accounts for 27% of the aggregate public-investment multiplier in the data, and 17% in the model. However, the same share would imply a much larger relevance of construction for the overall output response to public consumption, accounting for around 60% of the aggregate multiplier both in the data and the model.

This empirical evidence on the sectoral implications of fiscal expenditures validates both qualitatively and quantitatively the model predictions on the relationship between the high concentration of the sectoral contributions to public in-

[26] Appendix L directly compares the estimates of regressions (30) and (31) using both the real data and the simulated values from the model.

vestment and the relatively even distribution of its output gains across industries.

6 Conclusion

Aggregate and sectoral implications of public investment crucially depend on the interplay between the output elasticity to public capital and the presence of intermediate inputs. We use a production-network New Keynesian model to show that the socially optimal amount of public capital is twice as large as that predicted by the average one-sector economy, which leads to a substantial amplification of the public-investment multiplier. In addition, the model gives a novel prediction on how sectors react to public investment. Although public investment is concentrated in just a handful of industries, its effects are more evenly distributed across sectors than the ones of public consumption. We use linear projection methods on a panel of sectoral value added and aggregate public spending to validate this prediction in the data.

References

Acemoglu, Daron and Carvalho, Vasco and Ozdaglar, Asuman and Tahbaz-Salehi, Alireza. The network origins of aggregate fluctuations. *Econometrica*, 80(5):1977–2016, 2012.

Acemoglu, Daron and Akcigit, Ufuk and Kerr, William. Networks and the macroeconomy: An empirical exploration. *NBER Macroeconomics Annual*, 30(1):273–335, 2016.

Antolin-Diaz, Juan and Surico, Paolo. The long-run effects of government spending. *Mimeo*, 2022.

Atalay, Enghin. How important are sectoral shocks? *American Economic Journal: Macroeconomics*, 9(4):254–80, 2017.

Baqaee, David and Farhi, Emmanuel. Macroeconomics with heterogeneous agents and input-output networks. *Mimeo*, 2018.

Baqaee, David and Farhi, Emmanuel. The macroeconomic impact of microeconomic shocks: Beyond Hulten's theorem. *Econometrica*, 87(4):1155–1203, 2019.

Basu, Susanto and Fernald, John and Kimball, Miles. Are technology improvements contractionary? *American Economic Review*, 96(5):1418–1448, 2006.

Baxter, Marianne and King, Robert. Fiscal policy in general equilibrium. *American Economic Review*, 83(3):315–334, 1993.

Bigio, Saki and La'O, Jennifer. Distortions in production networks. *Quarterly Journal of Economics*, 135(4):2187–2253, 2020.

Blanchard, Olivier and Perotti, Roberto. An empirical characterization of the dynamic effects of changes in government spending and taxes on output. *Quarterly Journal of Economics*, 117(4):1329–1368, 2002.

Boehm, Christoph. Government consumption and investment: Does the composition of purchases affect the multiplier? *Journal of Monetary Economics*, 115:80–93, 2020.

Boehm, Christoph and Flaaen, Aaron and Pandalai-Nayar, Nitya. Input linkages and the transmission of shocks: Firm-level evidence from the 2011 Tōhoku earthquake. *Review of Economics and Statistics*, 101(1):60–75, 2019.

Bom, Pedro and Ligthart, Jenny. What have we learned from three decades of research on the productivity of public capital? *Journal of Economic Surveys*, 28(5):889–916, 2014.

Bouakez, Hafedh and Guillard, Michel and Roulleau-Pasdeloup, Jordan. Public investment, time to build, and the zero lower bound. *Review of Economic Dynamics*, 23:60–79, 2017.

Bouakez, Hafedh and Guillard, Michel and Roulleau-Pasdeloup, Jordan. The optimal composition of public spending in a deep recession. *Journal of Monetary Economics*, 114:334–349, 2020.

Bouakez, Hafedh and Rachedi, Omar and Santoro, Emiliano. The sectoral origins of the spending multiplier. *Mimeo*, 2022.

Bouakez, Hafedh and Rachedi, Omar and Santoro, Emiliano. The government spending multiplier in a multi-sector economy. *American Economic Journal: Macroeconomics*, 15(1):209–239, 2023.

Buera, Paco and Trachter, Nico. Sectoral development multipliers. *Mimeo*, 2023.

Cai, Chunbing and Roulleau-Pasdeloup, Jordan. The public investment multiplier: Insights from a new analytical framework. *Mimeo*, 2023.

Calvo, Guillermo. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398, 1983.

Chetty, Raj and Guren, Adam and Manoli, Day and Weber, Andrea. Does indivisible labor explain the difference between micro and macro elasticities? A meta-analysis of extensive margin elasticities. *NBER Macroeconomics Annual*, 27(1):1–56, 2013.

Clarida, Richard and Gali, Jordi and Gertler, Mark. Monetary policy rules and macroeconomic stability: Evidence and some theory. *Quarterly Journal of Economics*, 115(1):147–180, 2000.

Cox, Lydia and Müller, Gernot and Pasten, Ernesto and Schoenle, Raphael and Weber, Michael. Big G. *Mimeo*, 2022.

De Loecker, Jan and Eeckhout, Jan and Unger, Gabriel. The rise of market power and the macroeconomic implications. *Quarterly Journal of Economics*, 135(2):561–644, 2020.

Erceg, Christopher and Henderson, Dale and Levin, Andrew. Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics*, 46(2):281–313, 2000.

Erosa, Andrés and Fuster, Luisa and Kambourov, Gueorgui. Towards a microfounded theory of aggregate labour supply. *Review of Economic Studies*, 83 (3):1001–1039, 2016.

Fernald, John. A quarterly, utilization-adjusted series on total factor productivity. *Mimeo*, 2014.

Hall, Robert. By how much does GDP rise if the government buys more output? *Brookings Papers on Economic Activity*, pages 183–231, 2009.

Hobijn, Bart and Nechio, Fernanda. Sticker shocks: Using VAT changes to estimate upper-level elasticities of substitution. *Journal of the European Economic Association*, 17(3):799–833, 2019.

Horvath, Michael. Sectoral shocks and aggregate fluctuations. *Journal of Monetary Economics*, 45(1):69–106, 2000.

Ilzetzki, Ethan and Mendoza, Enrique and Végh, Carlos A. How big (small?) are fiscal multipliers? *Journal of Monetary Economics*, 60(2):239–254, 2013.

Jordà, Òscar. Estimation and inference of impulse responses by local projections. *American Economic Review*, 95(1):161–182, 2005.

Lanteri, Andrea. The market for used capital: Endogenous irreversibility and reallocation over the business cycle. *American Economic Review*, 108(9): 2383–2419, 2018.

Leduc, Sylvain and Wilson, Daniel. Roads to prosperity or bridges to nowhere? Theory and evidence on the impact of public infrastructure investment. *NBER Macroeconomics Annual*, 27(1):89–142, 2013.

Lee, Donghoon and Wolpin, Kenneth. Intersectoral labor mobility and the growth of the service sector. *Econometrica*, 74(1):1–46, 2006.

Leeper, Eric and Plante, Michael and Traum, Nora. Dynamics of fiscal financing in the United States. *Journal of Econometrics*, 156(2):304–321, 2010a.

Leeper, Eric and Walker, Todd and Yang, Shu-Chun. Government investment and fiscal stimulus. *Journal of Monetary Economics*, 57(8):1000–1012, 2010b.

Liu, Ernest. Industrial policies in production networks. *Quarterly Journal of Economics*, 134(4):1883–1948, 2019.

Malley, James and Philippopoulos, Apostolis. The macroeconomic effects of funding US infrastructure. *European Economic Review*, 152:104334, 2023.

Moro, Alessio and Rachedi, Omar. The changing structure of government consumption spending. *International Economic Review*, 63(3):1293–1323, 2022.

Nakamura, Emi and Steinsson, Jón. Five facts about prices: A reevaluation of menu cost models. *Quarterly Journal of Economics*, 123(4):1415–1464, 2008.

Pasten, Ernesto and Schoenle, Raphael and Weber, Michael. The propagation of monetary policy shocks in a heterogeneous production economy. *Journal of Monetary Economics*, 116:1–22, 2020.

Pedroni, Peter. Purchasing power parity tests in cointegrated panels. *Review of Economics and Statistics*, 83(4):727–731, 2001.

Proebsting, Christian. Market segmentation and spending multipliers. *Journal of Monetary Economics*, 128:1–19, 2022.

Ramey, Valerie. Identifying government spending shocks: It's all in the timing. *Quarterly Journal of Economics*, 126(1):1–50, 2011.

Ramey, Valerie. The macroeconomic consequences of infrastructure investment. In *Economic Analysis and Infrastructure Investment*, pages 219–276. University of Chicago Press, 2021.

Ramey, Valerie and Zubairy, Sarah. Government spending multipliers in good times and in bad: Evidence from US historical data. *Journal of Political Economy*, 126(2):850–901, 2018.

Vom Lehn, Christian and Winberry, Thomas. The investment network, sectoral comovement, and the changing US business cycle. *Quarterly Journal of Economics*, 137(1):387–433, 2022.